

The Factory Control System

Notation: MSB (Most-Significant-Byte or bit)
 LSB (Least-Significant-Byte or bit)

A control system for a conveyor belt uses three sensors to detect the movement of the belt (X), the presence or absence of items on the belt (Y) and the press of a stop button by the operator (Z). Each sensor X, Y and Z will send a 0 or a 1 to a 3-bit register with X as the MSB and Z as the LSB. Under the conditions corresponding to decimal values of 1, 2, 5 and 6, a buzzer will sound.

First, understand the question: There are three sensors

X - detect the movement of the belt

Y – detect the presence or absence of items on the belt

Z – detect the press of a stop button by the operator

Boolean values represent the two states of an event happening or an event not happening.

The decimal values that represent condition that should sound a buzzer are 1, 2, 5, and 6:

Decimal Values	Binary Values (XYZ)
1	001
2	010
5	101
6	110

a) Complete this truth table for this situation:

X	Y	Z	Buzzer
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

b) Now complete this Boolean expression for the conditions under which the buzzer will sound:

(not X • not Y • Z) +

The four conditions are for 001, 010, 101, and 110,
 001 010 101 110

$\overline{X}YZ$ $\overline{X}Y\overline{Z}$ $X\overline{Y}Z$ $XY\overline{Z}$


$(\text{not } X \cdot \text{not } Y \cdot Z) + (\text{not } X \cdot Y \cdot \text{not } Z) + (X \cdot \text{not } Y \cdot Z) + (XY \cdot \text{not } Z)$

c) Simplify the expression (you may use a Karnaugh Map):

$$\overline{X}YZ + \overline{X}Y\overline{Z} + X\overline{Y}Z + XY\overline{Z}$$

$$\left(\overline{X}YZ + \overline{X}Y\overline{Z}\right) + \left(X\overline{Y}Z + XY\overline{Z}\right)$$

Group expressions, then take out common terms

$$\overline{X}(YZ + Y\overline{Z}) + X(\overline{Y}Z + Y\overline{Z})$$


Notice that these terms are common to both of the main terms being + together: Factoring out this term:

$$(YZ + Y\overline{Z}) (\overline{X} + X)$$

always 1

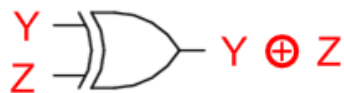
simplify:

$$(YZ + Y\overline{Z})$$

You may recognize this as Exclusive OR $Y \oplus Z$

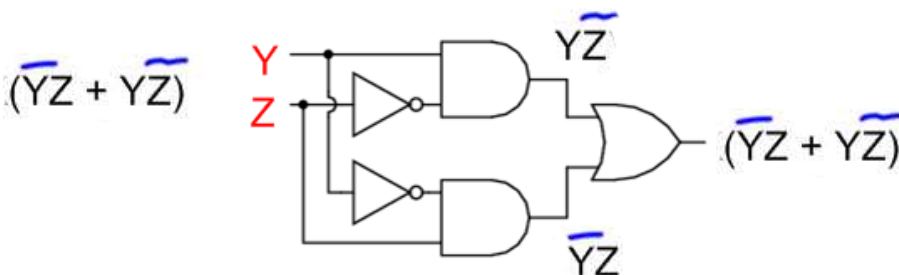
d) Draw an appropriate logic circuit

$$Y \oplus Z$$



Output triggers the buzzer

... is equivalent to ...



$$Y \oplus Z = (\overline{Y}Z + Y\overline{Z})$$